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NEW CONCEPT FUZZY SOFT SETS

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***Abstract*- In this research paper, we continue the study on fuzzy soft complements and investigate the properties and theorems of fuzzy soft dual point. Then discuss the fuzzy soft t-norms and fuzzy soft t-conorms. Which are important for further research on soft topology. This research not only can form the theoretical basis for further application of topology on soft sets but also lead to the development of information system.**

Keywords: Fuzzy soft sets, Fuzzy soft complements, Fuzzy soft dual point, Fuzzy soft t-norms, Fuzzy soft t-conorms.

I.INTRODUCTION

Fuzzy sets originated in a seminal paper by Lotfi A. Zadeh [10] in 1965.Since then, it has grown by laps and bounds and innumerable number of papers have appeared in various journals. But there exists difficulty, how to set the membership function in each particular case. The applications of fuzzy sets and fuzzy logic were ushered in by mamdani through a paper in 1975.The developments in applications were so dramatic that within 20 years both consumer products like cameras, washing machines, TV and industrial products, based on fuzzy logic Controllers, were rolled out in the market. The theory of fuzzy soft sets is more generalized concept than the theory of fuzzy soft sets but this theory has same difficulties. In 1999, Molodtsov [3] initiated the concept of soft sets. Molodtsov [3], is free of the difficulties present in this theories. The topological structures of soft set theories dealing with uncertainties were first studied by Cheang [2] .He introduces the notion of fuzzy topology and also studied some basic properties. K. Borgohain [6, 7,8] 2014 studied fuzzy soft separation axioms, fuzzy soft compact spaces. In 2016 K.Borgohain[9] gives to new concept of fuzzy soft topological space. Some other [6],[11],[12],[13][14] studied on the compact fuzzy soft topological space. My main aim in this research paper is to develop the basic properties of fuzzy soft special operations and establish several equivalent forms of fuzzy soft sets.

II. PRELIMINARIES

Definition 2.1[13]A pair (F,A) is called a fuzzy soft set over U where F:A $\rightarrow \tilde{P}$(U) is a mapping from a into $\tilde{P}$(U)

Definition2.2 [11] A pair (F, E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subset of the set U. In other words, the soft set is a parameterized family of subsets of the set U. Every set F(ε), $\varepsilon \in$ F from this family may be considered as the set of ε -elements of the soft set(F,E) or as the set of ε -approximate elements of the soft set.

Definition2.3 **[**10] Let U be an initial universe set and E be the set of parameters. Let $A \subseteq E$, a pair (F,A) is called a fuzzy soft set over U . Where F is a mapping given by $F : A \rightarrow I^U$, Where I^U denotes the collection of all fuzzy subset of U.

Definition 2.4[10] If T is a fuzzy soft topology on (U, E) , then (U,E,T) is said to be a fuzzy soft topological space . Also each member of T is called a fuzzy soft open set in (U, E, T)

Definition 2.5[13] The complement of a fuzzy soft set (F,A) is denoted by $(F,A)^C$ and is defined by

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$(F, A)^C = (F^C, A)$. Where $F^C: A \to \tilde{P}(U)$ is mapping given by

$F^C(\alpha) = U - F(\alpha) = [F(\alpha)]^C \ \forall \alpha \in A$

Definition 2.6[6]

Union of two fuzzy soft sets (F, A) and (G, B) over a common universe U is a fuzzy soft set (H,C) where C= A$\cup$B $\forall$ ε ∈ C

H (ε) = F(ε) if ε ∈ A-B, G (B) if ε ∈ B-A, F (ε) $\cup$ G (ε) if ε ∈ A $\cap$ B

And is written as(F,A) $\tilde{\cup}$ (G,B) = (H,C)

Definition 2.7[9]

Intersection of two fuzzy soft sets (F,A) and (G,B) over a common universe U is a fuzzy soft set (H,C), Where C= A $\cap$ B , ε ∈ C, H(ε)= F(ε) $\cap$G(ε) and is written as (F,A) $\cap$(G,B) = (H,C)

Definition 2.8[10] Let (X,E,T) be fuzzy soft topological space. A fuzzy soft set is called fuzzy soft closed if its complement is a member of $\tilde{T}$.

Definition2.9 [12] if T is a fuzzy soft topology on (X,E), the triple (X,E,T) is said to be fuzzy soft topological space. Each member of T is called fuzzy soft open set in (X,E,T).

Definition2.10 [9] Let (X,E,T) be a fuzzy soft topological space. Let (F,A) be fuzzy soft set over (X,E).Then the fuzzy soft closure of (F,A) denoted by $\overline{(F,A)}$ is defined as the intersection of all fuzzy soft closed sets which contains (F,A).

Definition2.11 [8] Let A$\tilde{\subset}$ E. Then the mapping $F_A: E \to \tilde{P}(U)$, defined by $F_A(e)=\mu^e F_A$ (a fuzzy subset of U) is called fuzzy soft over (U,E), Where $\mu^e F_A=\bar{0}$ if e∈E-A and $\mu^e F_A \neq \bar{0}$ if e∈A.

The set of all fuzzy soft set over (U,E) is denoted by FS(U,E)

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Definition2.12 [13] The fuzzy soft set FFS (U, E) is called null fuzzy soft set and is denoted by $\overline{\emptyset}$.

Here Fφ(e)= $\tilde{o}$ foe every e∈E

Definition2.13[8] Let (X,E,T) be a fuzzy soft topological space. Let (F,A) be a fuzzy soft set over (X,E). Then the fuzzy soft interior of (F,A) denoted by $(F,A)^o$, is defined as union of all fuzzy soft open sets contained in (F,A). That is $(F,A)^o = \tilde{\cup}\{(G,B):(G,B)$ is fuzzy soft open and $(G,B) \tilde{\subseteq} (F,A)\}$

III. MAIN RESULTS

Definition3.1 Any function to be considered a fuzzy soft complement it must satisfying at least the following two conditions
i)Fuzzy Soft Boundary condition: C(0)=1 and C(1)=0 , the function C is required to produce correct complement for fuzzy soft crisp sets.
ii)Fuzzy soft monotonicity : For all a, b ϵ[0,1], if a ≤ b then c(a) ≥ c(b) , where c is monotonic non – increasing
The condition i) & ii) are called the axiomatic skeleton for fuzzy soft complements.
Theorems3.1 Let a function C: [0,1]→ [0,1] is fuzzy soft monotonically non – increasing and involution function. Then, C is a continuous function and satisfy the boundary conditions. Moreover, C must be a fuzzy soft bijective function.
Proof: Since the range of C is [0,1], Therefore then c(0) ≤ 1 and c(1) ≥ 0
For c(0) ≤ 1
$\Rightarrow$C(c(0)) ≥ C(1)
$\Rightarrow$0≥ C(1)
Hence, c(0) ≤ 1and c(1) ≥ 0 $\Rightarrow$ c(1)=0
Again by fuzzy soft monotonic property
c(1) ≥ 0

$\Rightarrow$ C(c(1)) ≤ C(0)
$\Rightarrow$1 ≤ C(0)
Hence, c(0) ≤ 1and c(0) ≥ 1$\Rightarrow$ c(0)=1
Here C satisfies boundary conditions.
Now, we are to prove C is Fuzzy soft bijective function.
We know that a∈[0,1] then there exist b=C(a) ∈[0,1]
Such that c(b)=c(c(a))
$\Rightarrow$c(b)=a
Hence, C is onto function.
Let C($c(a_1)$)=$c(c(a_2))$
$\Rightarrow a_1=a_2$
C is also fuzzy soft one –one function. Hence C is bijective function.
Now to show that C is continuous function. Since is bijective and satisfies monotonicity. It cannot have any discontinuous points. To show that C has a fuzzy soft discontinuity at a_0.Then we have $b_0 \in [0,1]$ such that $b_0=\lim C(a) > C(a_0)$ and clearly, there must exist $b_1 \in [0,1]$ such that $b_0 > b_1 > C(a_0)$ for which no $a_1 \in [0,1]$ exists such that ($c(a_1)$)= b_1 . This conditions the fact that c is a bijective function. Thus C is a continuous function.
Definition3.2Fuzzy Soft Dual Point
If we are given a fuzzy soft complement C and a membership grade whose value is represented by a real number a $\in$[0,1] then any membership grade represented by the real number ${}^{d}a \in [0,1]$ such that
$C({}^{d}a) - {}^{d}a = a - c(a)$ is called a fuzzy soft dual point of a with respect to C.
Definition3.3Fuzzy Soft Decreasing Generator
A continuous and strictly decreasing function f: [0,1]→R such that f(1)=0 decreasing generator.
Definition3.4 Pseudo- Inverse of a Decreasing Generator

The pseudo-inverse of a decreasing generator f, denoted by f^{-1} is a function from R to [0,1] defined by f^{-1}(a)= i)1 for a $\in$ (0,1)(ii) f^{-1}(a) for a $\in$ [0,f(1)] (iii) 0 for a $\in$ (f(0),α) Where f^{-1} is the ordinary inverse of f.

Definition3.5 Fuzzy Soft Increasing Generator

A continuous and strictly decreasing function f: [0,1]→R such that f(0)=0 increasing generator.

Definition3.6 Pseudo- Inverse of a Increasing Generator

The pseudo-inverse of a Increasing generator g, denoted by g^{-1} is a function from R to [0,1] defined by g^{-1} (a)=i) 1 for a $\in$ (0,1) (ii) g^{-1} (a) for a $\in$ [0,f(1)] (iii)0 ,For a $\in$ (f(0),α) Where g^{-1} is the ordinary inverse of f.

Theorems3.2 Let**.** C: [0,1]→ [0,1] be a fuzzy soft function. Then, C is a fuzzy soft complement if and only if there exists a fuzzy soft continuous function g: [0,1]→R such that (i) g(0)=0 ,ii) g is strictly increasing and iii) c(a)= g^{-1}(g(1)-g(a)) for all aϵ[0,1]

Proof: we are to prove only if part. Let g be fuzzy soft continuous function from [0,1] to R such that it satisfies i), ii) and iii) conditions and we shall prove that C is a involutive fuzzy complement.

i)we now to show that c is fuzzy soft monotonic decreasing.

For any a, bϵ[0,1] if a< b then g(a)<g(b)

$\Rightarrow$- g(a)> -g(b)

$\Rightarrow$g(1) - g(a)>g(1)-g(b)

$\Rightarrow$ g^{-1}(g(1) - g(a))> g^{-1}(g(1)-g(b))

$\Rightarrow$c(a)> c(b)

Hence C satisfies the condition of monotonicity.

ii) Now we are to prove that C is involutive

For any aϵ[0,1]

C(c(a))= g^{-1}(g(1)-g(c(a))

$= g^{-1}$(g(1)-g(g^{-1}(g(1)-g(a))))
$= g^{-1}$(g(1)-g(1)+g(a))
$= g^{-1}$(g(a))
=a

Hence c is fuzzy soft involutive. It satisfies the all conditions. So C is a Fuzzy soft complement.

Example3.1 Prove that the increasing generator g(a)=a is determine a fuzzy soft complement.

Sol. Let g(a)=a=y

To find g^{-1}, We interchange the a and y then we get y=a and the inverse is g^{-1}(a)=a

Clearly, g(0)=0, g is continuous and strictly increasing, then c(a)= g^{-1} (g(1)-g(a))

Or c(a)= g^{-1}(1-a)

Hence c(a)=1-a

Theorems3.3. Let C: [0,1]→ [0,1] be a fuzzy soft function. Then, C is a fuzzy soft complement if and only if there exists a fuzzy soft continuous function f: [0,1]→R such that (i) f(1)=0 ,ii) g is strictly decreasing and iii) c(a)= f^{-1} (f(1)-f(a)) for all aϵ[0,1]

Proof: We know that a function C is fuzzy complement which involutive if there exist an increasing generator g such that

C(a)= g^{-1}(g(1) - g(a))

Now let f(a)= g(1) - g(a)

Then f(1)=0 and since g is strictly increasing , f is strictly decreasing.

Now we let f^{-1}(a)= g^{-1}(g(1)-a)

= g^{-1}(f(0)-a)

Since, f(0)=g(1)-g(0)=g(1)

Now replacing a by f^{-1}(a) then we get

f(f^{-1}(a))= g(1)-g (f^{-1}(a))

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= (g(1)-g(g^{-1}(g(1)-a))
=g(1)-g(1)+a
=a

Replacing a by f(a) then we get

f(f^{-1}(a))= g^{-1}(g(1)-f(a))
= g^{-1}(g(1)-(g(1)-g(a))
= g^{-1}(g(1)-g(1)+g(a))
= g^{-1}(g(a))=a

Again C(a)= g^{-1}(g(1)-g(a))
= f^{-1}(g(a))
= f^{-1}(g(1)-(g(1)-g(a))
= f^{-1}(f(0)-f(a))

Conversely, if a decreasing generator f is given, we can define an increasing generator g as g(a)=f(0)-f(a)

So we can write c(a)= f^{-1}(f(0)-f(a))
= g^{-1} (g(1)-g(a))

Hence, C is a Fuzzy soft complement.

Definition3.7 Fuzzy Soft Intersections

The intersection of two fuzzy sets A and B is a binary operation on the unit internal, that is a function of the form i: [0,1]× [0,1]→ [0,1]

For each xϵX, this function takes as its argument the pair consisting of the elements membership grades in set A and set B and the membership grade of the element in the set constituting A∩B(x)=i[A(x),B(x)], $\forall$ x$\in$X

A function i: [0,1]× [0,1]→ [0,1] is called a t-norm or fuzzy intersection if the following axioms are satisfied for all a, b, c, d ϵ[0,1]
Boundary condition i(a,1)=a
Monotonicity: b≤d ⇒ i(a,b) ≤i(a,d)
Commutativity: i(a,b)=i(b,a)
Associativity :i(a,i(b,d))=i(i(a,b),d)
Theorems3.4 The standard fuzzy intersection is the only idempotent fuzzy intersection.
Proof: Clearly min(a,a)=a for all a ϵ[0,1]
Assume that there exist a fuzzy intersection such that
I(a,a)=a , a ϵ[0,1]
Then for any a,b a ϵ[0,1]

i) a≤b then a=i(a,a) ≤i(a,b) ≤i(a,1)=a by monotonicity and boundary condition a ≤ i(a,b) ≤a
   hence i(a,b)=a

Therefore i(a, b)=a=min (a, b)

ii) Similarly if a≥b then b=i(b,b) ≤i(a,b) ≤i(1,b)=b
   And consequently i(a,b)=b=min (a,b)
   Hence i(a,b)=min (a,b) , for all a,b ϵ[0,1]

Other operations on Fuzzy sets.
Definition3.8 Disjunctive Sum
Disjunctive sum is the name of operation corresponding "Exclusive OR" logic.

i) Simple Disjunctive Sum
   We know that $\bar{A}(X)$ =1-A(x), $\bar{B}(X)$ =1-B(x)
   Then $(A \cap \bar{B})(x) = \min(A(x), 1 - B(x))$ and $(\bar{A} \cap B)(x) = \min(1 - A(x), B(x))$

Then simple disjunctive sum of two fuzzy sets A and B is defined by
$(A \oplus B)(x) = \max(\min(A(x), 1 - B(x)), \min(1 - A(x), B(x))$
Example3.2 Let A={(x_1,0.2),(x_2,0.7),(x_3,1.0),(x_4,0.0)}and B={(x_1,0.5),(x_2,0.3),(x_3,1.0),(x_4,0.1)}

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Sol: $\bar{A}$={(x_1,0.8),(x_2,0.3),(x_3,0.0),(x_4,1.0)}
$\bar{B}$={(x_1,0.5),(x_2,0.7),(x_3,0.0),(x_4,0.9)}
$A \cap \bar{B}$={(x_1,0.2),(x_2,0.7),(x_3,0.0),(x_4,0.0)}
$\bar{A} \cap B$={(x_1,0.5),(x_2,0.3),(x_3,0.0),(x_4,0.1)}
$A \oplus B$={(x_1,0.5),(x_2,0.7),(x_3,0.0),(x_4,0.1)}

IV. CONCLUSION

In twentieth century, mathematicians defined the concepts of sets and functions to represent problems. This way of representing problems in more grid. In many circumstances the solutions using the concept are meaningless. This difficulty was overcome by the fuzzy concept. Almost all the Mathematical, Engineering, Medicine, etc concepts have been redefined using the fuzzy sets. In this regards I have continued to study the properties of fuzzy soft complement. Introduced Fuzzy soft monotonicity, Fuzzy soft dual point ,increasing and decreasing generator etc and have established several interesting theorems, examples. I hope that the findings in this research work will help researchers enhance and promote the father study on fuzzy soft topology to carry out a general framework for their applications in practical life.

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